Basic Magneto Statics;

Multipoles:

Maxwell equations for time Varying fields,

Energy and Momentum :-

Introduction :- Magnetostatics is the study of magnetic fields in systems where the currents are steady. It is the magnetic analogue of electrostatics. The magnetization need not be static. The equations of Magnetostatics can be used to predict fast magnetic switching events that occur on time scales of nanoseconds or less.

Maxwell's equations for fixed charges or moving as a steady current $\vec{J}$ can be considered. the equations seperate into two equations for electric field and two for magnetic field. The magnetostatic equations for in both differential and integral forms are as follows

Gauss's law for magnetism $\quad \vec{\nabla}\cdot\vec{B} = 0$

$$\oint_S B\cdot dS = 0$$

Ampere's law $\quad \vec{\nabla}\times\vec{H} = J$

$$\oint_C H\cdot dl = I_{enc}$$

The condition ($V_i \simeq V_e \simeq V_d$) leads to a very important relationship between densities of different species.
As we know the continuity equation for electron

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e V_e) = 0$$

or $$\frac{\partial n_e}{\partial t} + n_e \nabla \cdot V_e = 0$$

or $$\frac{1}{n_e} \frac{\partial n_e}{\partial t} + \nabla \cdot V_e = 0$$

or $$\frac{1}{n_e} \frac{\partial n_e}{\partial t} = -\nabla \cdot V_e$$

multiplying and dividing above equation by n_{oe} (only L.H.S).

$$\left(\frac{n_{oe}}{n_e}\right) \frac{\partial n_e}{\partial t} \cdot \frac{1}{n_{oe}} = -\nabla \cdot V_e$$

or $$\boxed{\frac{\partial}{\partial t} \log \frac{n_e}{n_{oe}} = -\nabla \cdot V_e}$$

$\because \frac{\partial}{\partial t} \log \frac{n_e}{n_{oe}} = \frac{1}{n_e/n_{oe}} \frac{\partial}{\partial t} \frac{n_e}{n_{oe}}$

Similarly for ions and dust

$$\frac{\partial}{\partial t} \log \frac{n_i}{n_{oi}} = -\nabla \cdot V_i$$

$$\frac{\partial}{\partial t} \log \frac{n_d}{n_{od}} = -\nabla \cdot V_d$$

Since $V_e \approx V_i \approx V_d$
then

$$\log \frac{n_e}{n_{od}} \approx \log \frac{n_i}{n_{oi}} \approx \log \frac{n_d}{n_{od}}$$

which leads to

$$\boxed{\frac{n_e}{n_{oe}} = \frac{n_i}{n_{oi}} = \frac{n_d}{n_{od}}}$$

i.e number densities of different species are equal.

So expressing T_e, T_i, P_e and P_i in terms of dust density:

$$T_e = \left(\frac{n_d}{n_{od}}\right)^{2/3} T_{oe}$$

$$T_i = \left(\frac{n_d}{n_{od}}\right)^{2/3} T_{oi}$$

$$P_e = \left(\frac{n_d}{n_{od}}\right)^{5/3} P_{oe}$$

$$P_i = \left(\frac{n_d}{n_{od}}\right)^{5/3} P_{oi}$$

Also by stefan's Boltzmann law we know

$$P_{re} = \frac{\alpha_r}{3} T_e^4$$

Putting the value of T_e from above equation.

$$P_{re} = \frac{\alpha_1}{3}\left[\left(\frac{n_d}{n_{od}}\right)^{2/3} T_{oe}\right]^4$$

or

$$\boxed{P_{re} = \frac{\alpha_1}{3}\left(\frac{n_d}{n_{od}}\right)^{8/3} T_{oe}^4}$$

Similarly for ions i.e P_{ri}

$$\boxed{P_{ri} = \frac{\alpha_1}{3}\left(\frac{n_d}{n_{od}}\right)^{8/3} T_{oi}^4}$$

Putting the values of P_e, P_i, P_{re}, P_{ri} in eqs (5)

$$m_d n_d \frac{dV_d}{dt} = -\nabla\left[P_{oe}\left(\frac{n_d}{n_{od}}\right)^{5/3} + \frac{\alpha_1}{3} T_{oe}^4\left(\frac{n_d}{n_{od}}\right)^{8/3}\right.$$

$$\left. + P_{oi}\left(\frac{n_d}{n_{od}}\right)^{5/3} + T_{oi}^4\left(\frac{n_d}{n_{od}}\right)^{8/3}\right] + \frac{1}{c}(\vec{J}\times\vec{B}) - \rho_d\nabla\psi$$

Also from Maxwell's eqs we know.

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{\partial\vec{E}}{\partial t}$$

Due to quasi neutrality 2nd part on R.H.S is neglected

so $$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J}$$

or $$\boxed{\frac{c}{4\pi}(\vec{\nabla}\times\vec{B}) = \vec{J}}$$

So putting the value of $\vec{J}$ in equation of motion

$$m_d n_d \frac{dV_d}{dt} = -\nabla\left[(P_{oe}+P_{oi})\left(\frac{n_d}{n_{od}}\right)^{5/3} + \frac{\alpha_1}{3}\left(T_{oe}^4 + T_{oi}^4\right)\left(\frac{n_d}{n_{od}}\right)^{8/3}\right] + \frac{1}{c}\left[\frac{c}{4\pi}(\nabla\times B)\right]\times B - \rho_d \nabla\psi.$$

Also $m_d n_d = \rho_d$ (number density)

so
$$\rho_d \frac{dV_d}{dt} = -\nabla\left[(P_{oe}+P_{oi})\left(\frac{n_d}{n_{od}}\right)^{5/3} + \frac{\alpha_1}{3}\left(T_{oe}^4 + T_{oi}^4\right)\left(\frac{n_d}{n_{od}}\right)^{8/3}\right] + \frac{1}{4\pi}(\nabla\times B)\times B - \rho_d\nabla\psi$$

or
$$\frac{dV_d}{dt} = \frac{-\nabla}{\rho_d}\left[(P_{oe}+P_{oi})\left(\frac{n_d}{n_{od}}\right)^{5/3} + \frac{\alpha_1}{3}\left(T_{oe}^4 + T_{oi}^4\right)\left(\frac{n_d}{n_{od}}\right)^{8/3}\right] + \frac{1}{4\pi\rho_d}(\nabla\times B)\times B - \nabla\psi \quad \text{—— (6)}$$

Now from eqs (6) taking

$$\left(\frac{n_d}{n_{od}}\right)^{5/3} = \frac{n_d^{5/3}}{n_{od}^{5/3}} \quad \text{—— (a)}$$

Taking $n_d^{5/3}$ in linearized form.

$$n_d^{5/3} = n_{1d}^{5/3} + n_{od}^{5/3}$$